Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-09-15

Conceptual questions

Question 1.

(a) Warmup: Find a function $f(x)$ such that $f(0) = 3$, $f'(0) = 2$, and $f''(0) = 4$.

(b) Now onto MVC: Find a function $f(x, y)$ such that

$$f(0,0) = 3, f_x(0,0) = -2, f_y(0,0) = 0,$$
$$f_{xx}(0,0) = 7, f_{xy}(0,0) = -8, f_{yy}(0,0) = 3.$$

Question 2. Let $f(x, y)$ and $g(u, v)$ be two functions, related by

$$g(u, v) = f(e^u + \sin v, e^u + \cos v).$$

Use the following values to calculate $g_u(0,0)$ and $g_v(0,0)$ (not all of the below values may be relevant!).

$$f(0,0) = 3 \quad g(0,0) = 6 \quad f_x(0,0) = 4 \quad f_y(0,0) = 8$$
$$f(1,2) = 6 \quad g(1,2) = 3 \quad f_x(1,2) = 2 \quad f_y(1,2) = 5$$

Computations

Problem 1. Suppose that S is a (nice, differentiable...) surface which contains the two curves

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle,$$
$$\mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle.$$

Using this information, compute an equation of the tangent plane to S at the point $P(2, 1, 3)$.

Problem 2. The intersection of the plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse C. Find the tangent line to C at the point $(-1, -1, 2)$.

Problem 3.

(a) Suppose that $\mathbf{r}(t) = (x(t), y(t))$ is parametrized by arclength (recall that this means $|\mathbf{r}'(t)| = 1$; the particle is "moving at speed 1"). Show that the directional derivative of f in the direction of $\mathbf{r}'(t)$ is equal to $\frac{\mathrm{d}}{\mathrm{d}t}(f(\mathbf{r}(t)))$. Hint: Use the chain rule.

(b) Consider the function

$$f(x, y) = \cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right).$$

Using the preceding part, compute $f_y(1, 0)$. Hint: Use the unit circle.

Problem 4. Fix a nonnegative real number a and consider the function

$$f(x, y) = \begin{cases} \dfrac{(x + y)^a}{x^2 + y^2} & \text{if } (x, y) \neq (0,0), \\ 0 & \text{if } (x, y) = (0,0). \end{cases}$$

For what choices of a is the function f continuous?